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THE HECKSCHER-OHLIN THEORY OF FACTOR PRICE EQUALIZATION AND THE INDETERMINACY IN INTERNATIONAL SPECIALIZATION*

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1. INTRODUCTION

GIVEN THE PRODUCTION TECHNOLOGY, we may expect that the pattern of production and trade of a country would be determined by factor endowments and demand patterns in the Heckscher-Ohlin model. Imagine, however, a kind of perfect Heckscher-Ohlin world where the factor prices are equalized in every country. Suppose we are asked what the pattern of production and trade of a country would be, given the data on production technology, factor endowments and demand patterns of every country. If it is a world where the number of goods (n) exceeds the number of factors (m), we cannot say anything definite about the production and trade pattern of a country, i.e., the precise degree of international specialization is indeterminate. When $m < n$, the factor endowments and the demand patterns of each individual country have no formal places in the Heckscher-Ohlin theory of factor price equalization itself to determine the precise pattern of production and trade of each country. Furthermore, no one has yet rigorously explored the possible implications of the factor price equalization theorem specifically with respect to the determination of patterns of production and trade of each country.

The main purpose of this paper is to derive a theoretical framework which can specify the static global equilibrium pattern of specialization when n exceeds m on the basis of the Heckscher-Ohlin theory of factor price equalization. In order to specify the production and trade pattern of each country, we will introduce a simple assumption which seems reasonably realistic that, if the total value of outputs is the same, each country has a tendency to minimize international transaction activities. With this assumption it will be shown that we can eliminate the uncertainty concerning the precise pattern of production and trade of each country, and hence we can deduce a theoretical framework to determine the pattern of production and trade in multi-sectoral economy from the factor price equalization theorem.

2. DESCRIPTION OF THE MODEL

The Heckscher-Ohlin theory of factor price equalization, as proved by Samuelson [4], states that if all countries have the same homogeneous production functions of the first order with a nonsingular matrix of the factor intensities, and if complete specialization does not occur in any country, then the prices of the factors of production are completely equalized by interna-

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tional trade regardless of the factor endowments and the demand structures of the countries. McKenzie [3] adapts the activity analysis approach to the same factor price equalization problem, and the basic model of this paper is based on McKenzie's activity analysis.

A productive process is defined as the set of all sums of the basic activities in which a given list of inputs and output appears.[2] Any efficient activity can be the basis of the production for a sector. Since only relative quantities matter, because of linearity, it is convenient to normalize the activity vectors; we shall select the m-dimensional row vector a^i for which $\sum_{j=1}^m a_j^i = 1$ to represent the unit level of i-th productive activity ($i = 1 \cdots, n;\ j = 1, \cdots, m$). Then any vector of factor inputs belonging to the activity can be obtained by multiplying the normalized vector by the appropriate nonnegative constant q_i, i.e., $q_i a^i$ where $a^i = (a_1^i, \cdots, a_m^i)$. The n by m matrix of coefficients for ultimate factor inputs (a_j^i) is written as A.

Let an m-dimensional row vector f^k represent the factor endowments of the k-th trading country. We select the vector for which $\sum_{j=1}^m r_j^k = 1$ to represent the unit level of factor endowments in the k-th country. Then the vector of factor endowments f^k can be obtained by multiplying the normalized vector by the appropriate nonnegative constant B^k, i.e., $f^k = B^k r^k$ where $r^k = (r_1^k, \cdots, r_m^k)$.

Let S be the set of factor input vectors a^i for $i = 1, \cdots, n$ which satisfy the normalization rule $\sum_{j=1}^m a_j^i = 1$. And r^k, which represents the factor endowment ratios in the k-th country, is also normalized so that r^k is the element of S. All factor input vectors lie in S, and for a given set of good and factor prices (p, w) which is compatible with the profit conditions, we can consider the set K_{pw} of all elements of S which are expressible as positive linear combinations of a^i's coming from activities which earn zero profits at (p, w).[3] Here, z is in K_{pw} if and only if there are a^i, whose activities earn zero profits at (p, w) and $z = \sum_{i=1}^n t_i a^i$ for $0 \leqq t_i$ and $\sum_{i=1}^n t_i = 1$, where t_i are the levels at which the productive activities in use in the equilibrium are operated. If it is to be possible to exhaust the given vector of factor supplies of the k-th country, r^k must lie in K_{pw}. The points of K_{pw} which can be expressed as positive linear combinations of at least m linearly independent a^i's lie in the interior of K_{pw}. The competitive equilibrium with equalized factor prices implies that r^k lies in the interior of K_{pw} for all k, so that $r = \sum_{i=1}^n t_i a^i$ (for $0 \leqq t_i$ and $\sum_{i=1}^n t_i = 1$) in every country without specialization in any country. A country is said to be specialized if it uses fewer independent activities than it has ultimate factors.

[2] A productive activity may use both goods and factors as inputs; but the intermediate goods are converted into ultimate primary factors and hence here an activity is represented by a vector of primary inputs only. Absence of joint production is also assumed in this paper.

[3] Competitive equilibrium is characterized by the profit condition. Profits are defined for the i-th activity as $p_i q_i - \sum_j w_j f_j^i$. Here p is an n-dimensional column vector and w is m-dimensional column vector representing the prices of goods and factors respectively. It is assumed that anyone who governs an activity tries to maximize profits, but since the activities may be operated at any scale, if an activity is used, it must realize zero profits. Otherwise, there would be no limit to its size.

If the number of productive activities a^1 is equal to the number of factors (i.e., $n = m$), there is only one t vector in a country which can exhaust the given vector of factor supplies of the country, i.e., there is only one unique way r^k can be expressed as linear combinations of a^i's. If $m < n$, there are more than one set of good productions (q) which can exhaust the given factor supplies ($f^k = B^k r^k$) of the country at the given (p, w) and at the given coefficients of production. (Here q is an n-dimensional row vector such that $q^k = B^k t^k$ whose elements q_i^k representing the amount of i-th good produced in the k-th country.) Therefore, we can imagine many K_q's, the region of K_{pw} possibly spanned by the productive activities actually used in the country.

A given level of factor endowments (f^k) of a country implies a given level of income for the country. Suppose there is a given n-dimensional row vector of final demand for each good, $x = (x_1, \cdots, x_n)$, for a given level of income (Y) of a country. Here $Y = p \cdot q = p \cdot x$. We select the vector for which $\sum_{i=1}^{n} c_i = 1$ to represent the unit level of demand. That is, $x = B^\circ c$ where B° is a non-negative constant (for $0 \leqq c_i$ and $\sum_{i=1}^{n} c_i = 1$). And $r^\circ = \sum_{i=1}^{n} c_i a^i$ is an m-dimensional row vector which represents the factor supply ratios actually required to satisfy the domestic demand of the country. If there were no trade, r° should equal r. However, with trade, r° need not equal r; a country may assume any set of t_i which can exhaust r, i.e., $tA = r$, and then trade with other countries to obtain the collection of goods (x) whose factor proportions requirement can be expressed as r°, subject to the fixed maximum amount of income Y attainable in equilibrium with given factor endowment f. The competitive equilibrium with equalized factor prices implies that r° of each country is also in K_{pw}.

3. MINIMUM VALUE OF TRADE

Let $\bar{T}$ be an n-dimensional row vector such that $\bar{T} = x - q$. Then the $\bar{T}$ vector, which is determined by the difference between demand pattern and production pattern, represents the trade pattern of a country. Since there are more than one q vectors which can exhaust the given factor endowments of a country if $m < n$, even if we were given a definite demand pattern (x) corresponding to the specific factor endowments (f) of a country, we cannot have a definite idea about what production and trade pattern a country would assume.

Now we introduce the following assumption to determine the exact q (and hence the exact $\bar{T}$) of a country: transportation costs of goods would be small enough to allow us to neglect any difference between f.o.b. and c.i.f. prices of goods, but large enough to make consumers prefer a domestic good to its foreign counterpart when both goods are produced with the same inputs of factors. The consequence of this is the tendency to eliminate from trade those goods which are available through domestic production, domestic producers satisfying domestic consumption as much as possible with available productive resources. That is, we are assuming that each country tends to minimize its international transaction activities. With this assumption, we can show that there can be a unique global solution of the model with respect to the pattern of production and trade of each country.

In the factor price equalization theorem, regardless of the indeterminacy of international specialization, a set of imputed world good and factor prices emerges for the given production technology, factor endowments and demand structures of each country. We will have a set of (p, w) which will make a competitive equilibrium in the world possible.[4] Therefore, as will be shown, the argument can be reduced to a linear programming problem for the given set of (p, w) through the application of a Walras type tâtonnement process.

In the world as a whole, $\bar{r}^w = \bar{r}^{ow}$, where the m-dimensional row vectors $\bar{r}^w$ and $\bar{r}^{ow}$ represent the factor supply ratios of the world and the factor proportions actually required to satisfy the total world demand respectively; which implies

$$\sum_{k=1}^{N} \sum_{i=1}^{n} a^i t_i^k - \sum_{k=1}^{N} \sum_{i=1}^{n} a^i c_i^k = 0$$

where $k = 1, \cdots, N$ and N is the total number of countries.

Suppose an initial set of (t_i^k) which satisfies the profit condition at the equalized (p, w) in every country.[5] The set (t_i^k) is determined at random out of all feasible sets of (t_i^k) compatible with global competitive equilibrium and hence it may not minimize the trade value of each country. Suppose that a k-th country, say the first country, starts to adjust its t_i^1 to minimize its total absolute value of trade (V_1). We may regard it as the phase of preliminary groping towards the establishment of a global equilibrium, specified by the minimum-value-of-trade assumption. For the given initial t_i^k (where $k = 2, 3, \cdots, N$), the adjustment of t_i in the first country (and hence the change in the first country's trade pattern) implies that there would be positive excess demands or positive excess supplies of some (or all) goods in the international market. Now the expected rises or falls in the prices of goods would imply that adjustments in t_i^k (where $k = 2, 3, \cdots, N$) through the com-

[4] Competitive equilibrium is possible if the factor supplies can be exhausted by activities which meet the profit conditions.

[5] In this paper we are not concerned with the problem of the existence of a global equilibrium set of positive prices. We shall simply assume that such an equilibrium price vector exists for any configuration of demands and supplies, and begin our argument with the right set of prices (p, w) which make global competitive equilibrium possible. And hence we can avoid the problem arising from the possible changes in the parameters (for instance, changes in demand condition through changes in the distribution parameter) in the process of tâtonnement; and we can expect that the system will converge to equilibrium. In our process of tâtonnement in production, the only basic changes involved are those in the composition of products. Since the given data of a production economy are not the quantities of goods, but the quantities of productive factors, the tâtonnement does not affect the given data of world economy. Strictly interpreted, the groping for equilibrium which occurs here is prior to the conclusion of any trades. No production takes place until or unless the entire system is in equilibrium and until equilibrium is reached, there is no effective purchasing or selling of products either. Cf. Jaffé [2]. For the conditions under which the process of adjustment will converge to the equilibrium, refer to Uzawa [5], [6].

petitive market process are necessary. However, so long as the first country has made the adjustment in its t_i subject to the profit condition at the given initial set of (p, w) and also subject to its factor supply constraints $(r^1 = t^1A)$, the resulting set of prices determined by the whole adjusting processes will converge to the initial global equilibrium set of (p, w). We assume that the first country knows this ultimate result. Now let us assume that the first country has a new t vector (and as a result a specific trade pattern $\overline{T}^*$) which minimizes V_1. All other countries are assumed to have adjusted their t_i to allow the change in the first country, which implies

$$\sum_{k=2}^{N}\sum_{i=1}^{n} a^i t_i^k - \sum_{k=2}^{N}\sum_{i=1}^{n} a^i c_i^k = \left(\sum_{i=1}^{n} a^i c_i^1 - \sum_{i=1}^{n} a^i t_i^1\right)^*$$

where the asterisked parenthesis implies that each term in the parenthesis is fixed. The minimizing activity of the first country would result in the decrease, in absolute value, of the trade of the country by, say, dV_1. Since the exports of a country are the imports of other countries and vice versa, there would be a decrease in trade value in the other countries as a whole by the same amount dV_1.

Now suppose that the second country minimizes V_2 and again all the remaining $N-2$ countries make necessary adjustments for this change. Total value of world trade is further reduced by $2(dV_2)$. Suppose this process is continued until the $(N-1)$-th country. The N-th country adjusts its t_i to allow $(N-1)$-th country's minimizing activity. V_N is also minimized. Since every country has contributed the maximum magnitude of dV_k, the total value of world trade would be minimized. Now we have the static phase of gropings in which equilibrium is effectively established *ab ovo* as regards the level of production activities, under the stipulated minimum-value-of-trade condition and without any change in the data of the problem. The resulting equilibrium production and trade pattern of each country culminated by tâtonnement could be a unique one. The condition for uniqueness will now be examined.

The general problem presented so far is easily transformed to one which minimizes a linear form of nonnegative variables subject to a system of linear equations. That is, we can consider the problem in the following form: find the values of $t_1, \cdots, t_n$ for the k-th country which minimizes the linear form

$$V_k = \sum_{i=1}^{n} | p_i x_i - p_i q_i | = \sum_{i=1}^{n} | p_i B^\circ c_i - p_i B t_i | \tag{1}$$

subject to the condition that

$$t_i \geqq 0 \tag{2}$$

and

$$\begin{aligned} t_1 a_{11} + t_2 a_{12} + \cdots + t_n a_{1n} &= r_1 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ t_1 a_{m1} + t_2 a_{m2} + \cdots + t_n a_{mn} &= r_m \end{aligned} \tag{3}$$

where $p_i, a_{ij}, r_j, x_i, c_i, B$ and B° are given constants to the k-th country ($j = 1, \cdots, m$; $k = 1, \cdots, N$).

It is an established property of the linear programming problem that a sufficient condition is that every set of $(m + 1)$ vectors, defined by columns of coefficients in (1) and (3), be linearly independent so that the minimum feasible solution constructed from the given feasible solution is unique [1]. Since the factor price equalization theorem assumes the existence of certain m activities, whose factor input vectors $a^1, \cdots, a^m$ are linearly independent, all we have to assume here to assure a unique solution is that all productive activities have linearly independent vectors of factor inputs.

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REFERENCES

[1] DANTZIG, G. B., "Maximization of a Linear Function of Variables Subject to Linear Inequalities," in T. C. Koopmans ed., *Activity Analysis of Production and Allocation* (New York: John Wiley and Sons, 1951), 339-47.

[2] JAFFÉ, W., "Walras' Theory of Tâtonnement: A Critique of Recent Interpretations," *The Journal of Political Economy*, LXXV (February, 1967), 1-19.

[3] MCKENZIE, L. W., "Equality of Factor Prices in World Trade," *Econometrica*, XXIII (July, 1955), 239-57.

[4] SAMUELSON, P. A., "Prices of Factors and Goods in General Equilibrium," *Review of Economic Studies*, XXI (October, 1953), 1-20.

[5] UZAWA, H., "The Stability of Dynamic Process," *Econometrica*, XXIX, (October, 1961), 617-31.

[6] ———, "Walras' Tâtonnement in the Theory of Exchange," *Review of Economic Studies*, XXVII (June, 1960), 182-94.